



ELSEVIER

4 October 2001

PHYSICS LETTERS B

Physics Letters B 517 (2001) 355–362

www.elsevier.com/locate/npe

# Effects of periodic matter in kaon regeneration

Evgeny Akhmedov <sup>a,1</sup>, Augusto Barroso <sup>b</sup>, Petteri Keränen <sup>b</sup>

<sup>a</sup> *Institut für Theoretische Physik T30, Physik Department, Technische Universität München, James-Frank-Straße, D-85748 Garching b. München, Germany*

<sup>b</sup> *Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Av. Prof. Gama Pinto 2, P-1649-003 Lisboa, Portugal*

Received 26 July 2001; accepted 16 August 2001

Editor: P.V. Landshoff

## Abstract

We study the effects of periodic matter in kaon regeneration, motivated by the possibility of parametric resonance in neutrino oscillations. The large imaginary parts of the forward kaon–nucleon scattering amplitudes and the decay width difference  $\Delta\Gamma$  prevent a sizable enhancement of the  $K_L \rightarrow K_S$  transition probability. However, some interesting effects can be produced using regenerators made of alternating layers of two different materials. Despite the fact that the regenerator has a fixed length one can obtain different values for the probability distribution of the  $K_L$  decay into a final state. Using a two-arm regenerator set up it is possible to measure the imaginary parts of the  $K^0(\bar{K}^0)$ –nucleon scattering amplitudes in the correlated decays of the  $\phi$ -resonance. Combining the data of the single-arm regenerator experiments with direct and reverse orders of the matter layers in the regenerator one can independently measure the CP violating parameter  $\delta$ .

© 2001 Elsevier Science B.V. Open access under [CC BY license](#).

PACS: 11.30.Er; 13.75.Jz

Keywords: Kaon oscillations; Regeneration

Recently, there has been a renewed interest [1] in the possibility of parametric resonance in neutrino oscillations in matter suggested in [2,3]. For a neutrino beam propagating in a medium with periodic density, one can obtain a large probability for the transition from one flavour state to another, even if the neutrino mixing angles both in vacuum and in matter are small.

In nature, there are other systems similar to oscillating neutrinos, in particular the neutral mesons  $K^0$ – $\bar{K}^0$ . Hence, it is interesting to investigate if one can

obtain the parametric resonance in this case. The analogue of the neutrino weak flavour basis are  $K^0$  and  $\bar{K}^0$  and the mass eigenstates are  $K_L$  and  $K_S$ . Since the former states are maximally mixed, it is obvious that one cannot enhance the  $K^0$ – $\bar{K}^0$  transition probability. However, in this case this is not the relevant question.

Let us assume that we have a neutral kaon beam propagating in vacuum. After a time  $t$  larger than the  $K_S$  lifetime ( $\tau_S = 0.894 \times 10^{-10}$  s) the beam is essentially a  $K_L$  beam. If this beam traverses a thin slab of material, a small  $K_S$  component will emerge, because  $K^0$  and  $\bar{K}^0$  have different scattering amplitudes. This is the well-known regeneration phenomenon (see, e.g., [4]). Assuming that the beam enters the regenerator at  $t = 0$  and denoting by  $|K_R(t)\rangle$  the state of the beam

*E-mail addresses:* akhmedov@physik.tu-muenchen.de (E. Akhmedov), barroso@cii.fc.ul.pt (A. Barroso), keranen@cii.fc.ul.pt (P. Keränen).

<sup>1</sup> On leave from National Research Centre Kurchatov Institute, Moscow 123182, Russia.

when it emerges, we have

$$|K_R(t)\rangle = \langle \tilde{K}_S | T | K_L \rangle |K_S\rangle + \langle \tilde{K}_L | T | K_L \rangle |K_L\rangle, \quad (1)$$

where  $\langle \tilde{K}_{S,L} | T | K_L \rangle$  are the transition amplitudes in the regenerator, and  $\langle \tilde{K}_{S,L} |$  are the reciprocal states (see Eqs. (6) and (7) below). If the regenerator is a medium with a density that is a periodic function of the coordinate along the beam direction, we would like to see if it is possible to enhance the  $K_L \rightarrow K_S$  transition amplitude. Our aim in this Letter is to address this question.

Assuming CPT conservation, but not CP conservation, the rest-frame evolution equation for the  $K^0$ – $\bar{K}^0$  system propagating in a medium is

$$i \frac{d}{dt} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = \begin{pmatrix} \mu + V & \frac{p}{q} \frac{\Delta\mu}{2} \\ \frac{q}{p} \frac{\Delta\mu}{2} & \mu + \bar{V} \end{pmatrix} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}, \quad (2)$$

where  $t$  is the proper time (we follow closely the notation of Ref. [4]). Hence, in vacuum ( $V = \bar{V} = 0$ ) the eigenstates of the Hamiltonian  $H_0$  are

$$|K_L\rangle = p|K^0\rangle + q|\bar{K}^0\rangle \quad (3)$$

and

$$|K_S\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \quad (4)$$

with the corresponding eigenvalues  $\mu_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S}$ ,  $\mu = (\mu_L + \mu_S)/2$  and  $\Delta\mu = \mu_L - \mu_S$ . Since the phase of  $p/q$  is of no physical significance, we shall assume this ratio to be real.<sup>2</sup> We write

$$p = \sqrt{\frac{1+\delta}{2}}, \quad q = \sqrt{\frac{1-\delta}{2}}, \quad (5)$$

where  $\delta \simeq 3 \times 10^{-3}$  [5] is a measure of CP violation. Since CP is not conserved, the diagonalization of  $H_0$  cannot be accomplished with a unitary transformation. This, in turn, implies the use of the reciprocal basis [6]

$$\langle \tilde{K}_L | = \frac{1}{2} \left( \frac{1}{p} \langle K^0 | + \frac{1}{q} \langle \bar{K}^0 | \right), \quad (6)$$

$$\langle \tilde{K}_S | = \frac{1}{2} \left( \frac{1}{p} \langle K^0 | - \frac{1}{q} \langle \bar{K}^0 | \right). \quad (7)$$

In a medium with  $N_a$  nuclei per unit volume,  $V$  ( $\bar{V}$ ) is given in terms of the forward scattering amplitude

$f(0)$  ( $\bar{f}(0)$ ) for a  $K^0$  ( $\bar{K}^0$ ) beam [4], i.e.,

$$V = -\frac{2\pi N_a}{m} f(0), \quad (8)$$

with the average kaon mass  $m = 2.52 \text{ fm}^{-1}$ . The simplest way to introduce a periodic medium is to consider two different elements with number densities  $N_a$  and  $N_b$  positioned one after the other and to build a regenerator with  $\kappa$  layers of this  $ab$  junction. The beam evolution through this multilayer regenerator can be described in terms of the evolution operator

$$U_\kappa = \underbrace{U_b U_a \times \cdots \times U_b U_a}_\kappa, \quad (9)$$

with

$$U_a = \exp(-i H_a t_a) \quad (10)$$

and

$$U_b = \exp(-i H_b t_b), \quad (11)$$

where  $H_i$  ( $i = a, b$ ) are the Hamiltonians for layers  $a$  or  $b$ , given in Eq. (2). Since this is a  $2 \times 2$  matrix it is convenient to represent it using the Pauli  $\sigma$  matrices. One can then write

$$H_a = F_a + \sigma \mathbf{E}_a, \quad (12)$$

where

$$F_a = \mu + \frac{V_a + \bar{V}_a}{2}, \quad (13)$$

and  $\mathbf{E}_a$  is a three dimensional vector with components

$$E_a^{(1)} = \frac{\Delta\mu}{4} \left( \frac{p}{q} + \frac{q}{p} \right), \quad (14)$$

$$E_a^{(2)} = i \frac{\Delta\mu}{4} \left( \frac{p}{q} - \frac{q}{p} \right), \quad (15)$$

$$E_a^{(3)} = \frac{V_a - \bar{V}_a}{2} \equiv \frac{\Delta V_a}{2}, \quad (16)$$

which are complex numbers. Introducing the complex unit vector

$$\mathbf{n}_a = \frac{\mathbf{E}_a}{E_a}, \quad E_a \equiv \sqrt{\mathbf{E}_a \cdot \mathbf{E}_a}, \quad (17)$$

and

$$\varphi_a = E_a t_a, \quad (18)$$

<sup>2</sup> Since the relative phases between  $p/q$  and certain ratios of amplitudes do have physical content, the phase convention we use here implies a phase convention for the decay amplitudes as well.

one immediately obtains

$$U_a = \exp(-iF_a t_a)(\cos \varphi_a - i\boldsymbol{\sigma} \cdot \mathbf{n}_a \sin \varphi_a). \quad (19)$$

With the obvious replacements  $a \rightarrow b$  one obtains from Eq. (19)  $U_b$ . Then the product  $U_b U_a$  is

$$U_b U_a = \exp[-i(F_a t_a + F_b t_b)][Y - i\boldsymbol{\sigma} \cdot \mathbf{X}], \quad (20)$$

with

$$Y = \cos \varphi_a \cos \varphi_b - \sin \varphi_a \sin \varphi_b (\mathbf{n}_a \cdot \mathbf{n}_b), \quad (21)$$

$$\mathbf{X} = \sin \varphi_a \cos \varphi_b \mathbf{n}_a + \sin \varphi_b \cos \varphi_a \mathbf{n}_b - \sin \varphi_a \sin \varphi_b (\mathbf{n}_a \times \mathbf{n}_b). \quad (22)$$

The vectors that we have introduced have complex components. However, the dot products, such as  $\mathbf{n}_a \cdot \mathbf{n}_b$ , must be simply understood as

$$\mathbf{n}_a \cdot \mathbf{n}_b = \sum_i^3 n_a^{(i)} n_b^{(i)}. \quad (23)$$

Notice that the third component of  $\mathbf{n}_a \times \mathbf{n}_b$  is identically zero. Then Eq. (22) shows that  $X^{(3)}$  is symmetric with respect to the interchange of  $a$  and  $b$ . On the other hand, Eq. (15) shows that  $n_a^{(2)}$  and  $n_b^{(2)}$  vanish in the limit of CP conservation. Then, in this limit,  $X^{(2)}$  is antisymmetric in  $a$  and  $b$ . Furthermore, in the same approximation the first component of  $\mathbf{n}_a \times \mathbf{n}_b$  is also zero. Hence  $X^{(1)}$  is symmetric with respect to the interchange of  $a$  and  $b$ .

A straightforward calculation shows that  $Y^2 + \mathbf{X} \cdot \mathbf{X} = 1$ . Then, defining another complex angle  $\Phi$  such that

$$\cos \Phi = Y, \quad (24)$$

$$\sin \Phi = \sqrt{\mathbf{X}^2}, \quad (25)$$

it is possible to rewrite Eq. (20) in the form

$$U_b U_a = \exp[-i(F_a t_a + F_b t_b)] \exp(-i\boldsymbol{\sigma} \cdot \hat{\mathbf{X}} \Phi) \quad (26)$$

with

$$\hat{\mathbf{X}} = \frac{\mathbf{X}}{\sqrt{\mathbf{X}^2}}. \quad (27)$$

This evolution operator is written in the  $K^0$ – $\bar{K}^0$ -basis. Denoting  $U_{ab} \equiv U_b U_a$ , the symmetry properties of  $X^{(i)}$  deduced above enables us to obtain

$$\langle K^0 | U_{ab} | K^0 \rangle = \langle K^0 | U_{ba} | K^0 \rangle, \quad (28)$$

$$\langle \bar{K}^0 | U_{ab} | \bar{K}^0 \rangle = \langle \bar{K}^0 | U_{ba} | \bar{K}^0 \rangle, \quad (29)$$

but

$$\langle K^0 | U_{ab} | \bar{K}^0 \rangle - \langle \bar{K}^0 | U_{ba} | K^0 \rangle \propto \delta, \quad (30)$$

$$\langle \bar{K}^0 | U_{ab} | K^0 \rangle - \langle K^0 | U_{ba} | \bar{K}^0 \rangle \propto \delta, \quad (31)$$

i.e., the difference vanishes if CP is conserved.

Finally, the evolution matrix for the propagation through  $\kappa$   $ab$ -layers is simply

$$U_\kappa = \exp[-i\kappa(F_a t_a + F_b t_b)] \times (\cos(\kappa\Phi) - i\boldsymbol{\sigma} \cdot \hat{\mathbf{X}} \sin(\kappa\Phi)). \quad (32)$$

Inserting  $U_\kappa$  between the appropriate bra- and ket-vectors given by Eqs. (3), (4) and (6), (7) respectively, one obtains the  $K_L \rightarrow K_L$  and the  $K_L \rightarrow K_S$  transition amplitudes

$$\begin{aligned} \langle \tilde{K}_L | U_\kappa | K_L \rangle &= \exp[-i\kappa(F_a t_a + F_b t_b)] \\ &\times \left\{ \cos(\kappa\Phi) - \frac{i}{2} \sin(\kappa\Phi) \right. \\ &\times \left( \left[ \frac{p}{q} + \frac{q}{p} \right] \hat{X}_1 + i \left[ \frac{p}{q} - \frac{q}{p} \right] \hat{X}_2 \right) \Big\} \end{aligned} \quad (33)$$

and

$$\begin{aligned} \langle \tilde{K}_S | U_\kappa | K_L \rangle &= \exp[-i\kappa(F_a t_a + F_b t_b)] \\ &\times \frac{i}{2} \sin(\kappa\Phi) \left\{ -2\hat{X}_3 + \left[ \frac{p}{q} - \frac{q}{p} \right] \hat{X}_1 \right. \\ &\left. + i \left[ \frac{p}{q} + \frac{q}{p} \right] \hat{X}_2 \right\}. \end{aligned} \quad (34)$$

Let us start our discussion with a careful examination of Eqs. (14)–(16). The vectors  $\mathbf{E}_a$  and  $\mathbf{E}_b$  have the first components proportional to  $\Delta\mu/2$  and the third components proportional to  $\Delta V_i/2$ . These quantities  $\Delta\mu$  and  $\Delta V$  play a crucial role in the effect that we are searching for. On the contrary, the mean values  $\mu$  and  $(V_i + \bar{V}_i)/2$  are far less important. Their real parts disappear when we take the modulus square of the amplitude to obtain the transition probabilities and their imaginary parts give the overall damping factors.

As a first approximation, we neglect CP violation. Let us further assume that  $\Delta\mu$  and  $\Delta V_i$  are real. A real  $\Delta\mu$  means that  $\Delta\Gamma = 0$ . Although this is not true for the  $K$ -meson system, there is no fundamental reason why it could not be so. Indeed, such a situation occurs closely for the  $B^0$ – $\bar{B}^0$  mesons. A real  $\Delta V_i$  implies equal imaginary parts for the  $K^0$  and  $\bar{K}^0$  forward scattering amplitudes. As it is well known, this is not

the case. This is in contrast with the case of neutrinos, where the absorption is weak, and to the leading order in weak interaction the scattering amplitudes are real.

Within this unrealistic approximation it is possible to achieve a parametric resonance in  $K_L \leftrightarrow K_S$  transitions in matter. The parametric resonance condition is  $X_3 = 0$  [7]; we shall consider a particular realization of this condition in which the times  $t_a$  and  $t_b$  are chosen such that  $\cos \varphi_a = \cos \varphi_b = 0$ . Then it follows from Eq. (22) that

$$\mathbf{X} = \pm(\mathbf{n}_a \times \mathbf{n}_b). \quad (35)$$

As described above, the third component of the cross product  $\mathbf{n}_a \times \mathbf{n}_b$  is identically zero and if one neglects CP violation the first component is also zero. In this approximation Eqs. (33) and (34) become

$$\langle \tilde{K}_L | U_\kappa | K_L \rangle = \exp[-i\kappa(F_a t_a + F_b t_b)] \cos(\kappa \Phi) \quad (36)$$

and

$$\langle \tilde{K}_S | U_\kappa | K_L \rangle = -\exp[-i\kappa(F_a t_a + F_b t_b)] \sin(\kappa \Phi) \hat{X}_2. \quad (37)$$

For an appropriate number of layers,  $\kappa$ , one can suppress the  $K_L \rightarrow K_L$  probability and, at the same time, enhance the  $K_L \rightarrow K_S$  transition probability. To illustrate this effect, we plot the  $K_L \rightarrow K_S$  transition probability as a function of  $\kappa$  in Fig. 1. The calculation was done for a regenerator made of  $^{27}\text{Al}$  and  $^{184}\text{W}$  and for an initial  $K_L$  beam obtained from the decay of the  $\phi$  resonance at rest. The values of the  $K^0$  and  $\bar{K}^0$  scattering amplitudes on protons and neutrons were taken from Ref. [8]. As we have explained,  $\Delta\Gamma$  and the imaginary parts of  $\Delta V_i$  were set equal to zero.

The times  $t_a$  and  $t_b$  were chosen in such a way that a complete  $K_L \rightarrow K_S$  conversion could be obtained. If we move away from this resonance condition we still obtain an oscillatory  $K_L \rightarrow K_S$  transition probability  $P(K_L \rightarrow K_S)$  but with a smaller maximal conversion. For instance, decreasing both  $t_a$  and  $t_b$  by 17% reduces the maximum value of  $P(K_L \rightarrow K_S)$  from 1 to 0.145.

The resonance values of  $t_a$  and  $t_b$  ( $59.210 \times 10^{-11}$  s and  $57.289 \times 10^{-11}$  s) are a factor of seven or six larger than  $\tau_S$ . This by itself is sufficient to explain that the effect disappears as soon as we introduce the right values of  $\Gamma$  and  $\Delta\Gamma$ , even with  $\text{Im}(V) = \text{Im}(\bar{V}) = 0$ . We have checked that, in this case,  $P(K_L \rightarrow K_S) \simeq 10^{-4}$  for  $\kappa = 1$  and decreases slowly with  $\kappa$ . In addition, if we introduce the correct values for the

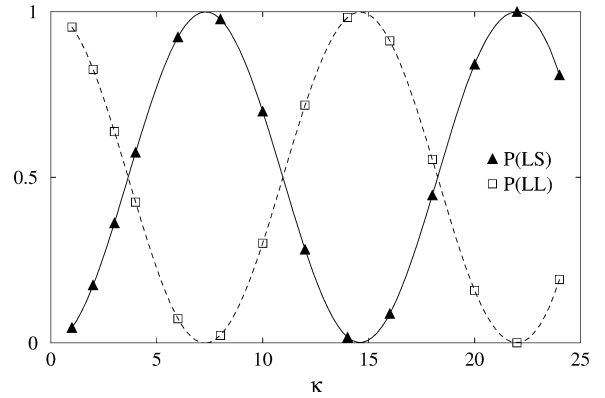


Fig. 1.  $K_L \rightarrow K_S$  and  $K_L \rightarrow K_L$  transition probabilities  $P(LS)$  and  $P(LL)$ , respectively, with  $\text{Im}(V) = \text{Im}(\bar{V}) = \Delta\Gamma = \Gamma = 0$ . The regenerator is  $^{27}\text{Al}$ – $^{184}\text{W}$  and  $t_a = 59.21 \times 10^{-11}$  s and  $t_b = 57.29 \times 10^{-11}$  s.

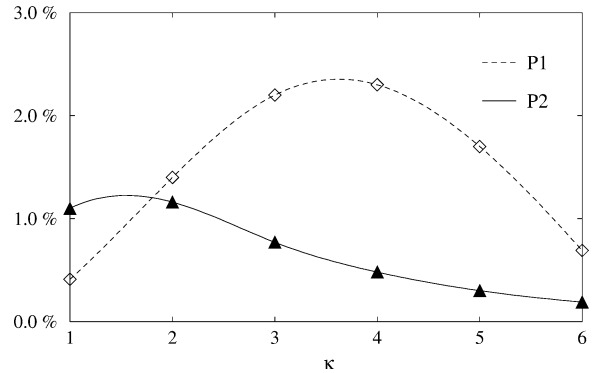


Fig. 2.  $P_2$  is the  $K_L \rightarrow K_S$  transition probability for  $^{27}\text{Al}$ – $^{184}\text{W}$  regenerator with  $t_a = t_b = 8 \times 10^{-11}$  s.  $P_1$  is the same probability when neglecting all imaginary parts.

imaginary parts of the scattering amplitudes,  $P(K_L \rightarrow K_S)$  for  $\kappa = 1$  is further reduced to  $8 \times 10^{-5}$  and even  $P(K_L \rightarrow K_L)$  becomes 0.05, while in the previous case it was 0.93.

Clearly, any measurable effect with kaons propagating in matter requires times of the order of  $\tau_S$ . Unfortunately, for such times, even the toy model without imaginary parts gives a maximum value for  $P(K_L \rightarrow K_S)$  of the order of 0.02 only. Then, the damping due to the imaginary parts washes out the effect. This is shown in Fig. 2 where we compare for the same  $t_a$  and  $t_b$   $P(K_L \rightarrow K_S)$  in the toy model ( $P_1$ ) and for real kaons traversing a real  $^{27}\text{Al}$ – $^{184}\text{W}$  regenerator.

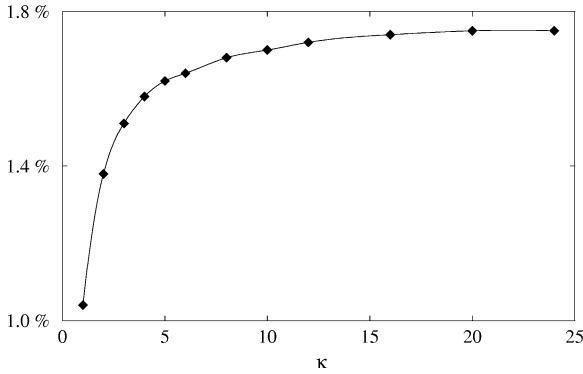


Fig. 3.  $K_L \rightarrow K_S$  transition probability for  $^{27}\text{Al}$ – $^{184}\text{W}$  regenerator with  $t_a = t_b = \frac{12}{\kappa} \times 10^{-11}$  s.

So far, in all cases that we have considered, the total time that the particles spend in the regenerator,  $t = \kappa(t_a + t_b)$ , increases linearly with  $\kappa$ . Obviously, after a few layers most of the particles will disappear due to their decay or absorption. Hence, it is interesting to examine another type of experiment, where the total time  $t$  is kept fixed, i.e., as  $\kappa$  increases the times  $t_a$  and  $t_b$  are proportionally reduced. In Fig. 3 we plot  $P(K_L \rightarrow K_S)$  as a function of  $\kappa$  for this situation. For the beam velocity that we are considering,  $10^{-11}$  s corresponds to a path length of the order of 1 mm in vacuum. Then from Fig. 3 one can see that a regenerator made of a 12 mm layer of  $^{27}\text{Al}$  followed by another layer of 12 mm of  $^{184}\text{W}$  ( $\kappa = 1$ ) is less efficient than another regenerator with four alternating  $^{27}\text{Al}$ – $^{184}\text{W}$  layers of 6 mm each ( $\kappa = 2$ ). Perhaps this effect is better illustrated if, instead of the transition probability, we consider the decay of the kaons into a final state  $f$  after traversing the regenerator. From Eq. (1) one can calculate the time distribution  $P(K_R(t) \rightarrow f)$  of the final state  $f$  after the kaon state initially produced as  $K_L$  passes through the regenerator and then spends outside it the proper time  $t$  (which for simplicity we took equal to the proper time spent inside the regenerator). The result is (e.g., Ref. [4])

$$P(K_R(t) \rightarrow f) = |\langle f | T | K_S \rangle|^2 |\langle \tilde{K}_S | U_\kappa | K_L \rangle|^2 \times [e^{-\Gamma_S t} + |v_f|^2 e^{-\Gamma_L t} + 2|v_f| e^{-\Gamma t} \cos(\theta_f - \Delta m t)], \quad (38)$$

where

$$v_f = |v_f| e^{i\theta_f} \equiv \frac{\langle \tilde{K}_L | U_\kappa | K_L \rangle}{\langle \tilde{K}_S | U_\kappa | K_L \rangle} \eta_f. \quad (39)$$

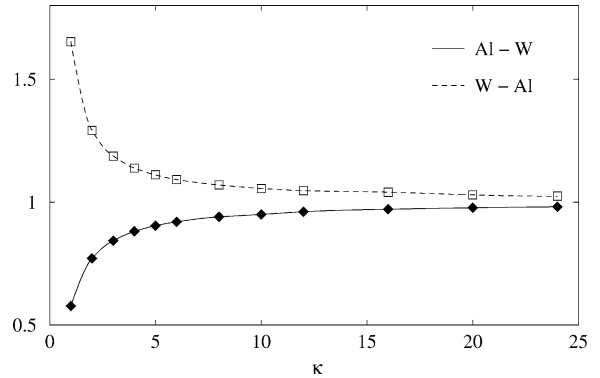


Fig. 4.  $P(K_R \rightarrow \pi^+\pi^-)/(10^7 \text{ s}^{-1})$  for two regenerators (Al–W or W–Al) with  $t_a = t_b = \frac{12}{\kappa} \times 10^{-11}$  s.

In our example, shown in Fig. 4, we have assumed that one measures the  $\pi^+\pi^-$  final state. The magnitude and the phase of  $\eta_{+-}$  were taken from Ref. [5]. The probability distribution after passing  $\kappa$  ( $^{27}\text{Al}$ – $^{184}\text{W}$ ) layer junctions increases with  $\kappa$ . In the same Fig. 4 we also plot the probability distribution for a regenerator where the layers are in reverse order. In this case  $P(K_R \rightarrow f)$  decreases with  $\kappa$ , and both curves tend to a common limit. This is easy to understand. As the number of layers increase we are effectively approaching a “mixed material” with a density that has the average density of aluminum and tungsten. Since the regeneration effect is proportional to the density of the regenerator, one can understand that  $P(K_R \rightarrow \pi^+\pi^-)$  increases with  $\kappa$  for the  $^{27}\text{Al}$ – $^{184}\text{W}$  regenerator and decreases in the  $^{184}\text{W}$ – $^{27}\text{Al}$  case.<sup>3</sup> The variation with the order of the layers (notice that their total number is fixed) is a nice example of quantum mechanics interference. In this problem, the evolution matrix for each individual layer (cf. Eqs. (10), (11)) is an element of the  $U(2)$  group. Hence, the evolution for the total number of layers is, of course, an element of  $U(2)$ . Since  $U(2)$  is a non-Abelian group, shuffling the layers one obtains a different evolution operator. From this point of view, Fig. 4 is a consequence of the non-commutativity of the  $U(2)$  group.

<sup>3</sup> For  $\kappa = 1$  the regeneration in the  $^{27}\text{Al}$ – $^{184}\text{W}$  case is less efficient than in the  $^{184}\text{W}$ – $^{27}\text{Al}$  one because a fraction of  $K_L$  decays in aluminum before they reach a more efficient regenerator — tungsten.

One should realize that the results shown in Fig. 4 are independent of the CP-violating parameter  $\delta$ . However, it is possible to use this type of regenerators to measure CP violation at the  $\phi$ -factories. To see how the effect arises let us recall that the  $\phi$ -meson decays into the antisymmetric combination

$$\frac{1}{\sqrt{2}}[|K^0(\mathbf{p})\rangle|\bar{K}^0(-\mathbf{p})\rangle - |\bar{K}^0(\mathbf{p})\rangle|K^0(-\mathbf{p})\rangle], \quad (40)$$

where  $\mathbf{p}$  denotes the momentum of the particle. We assume that in the direction of  $-\mathbf{p}$  we have a detector, called “left”, and in the direction of  $\mathbf{p}$  another detector, called “right”. Both detectors measure muons from the semileptonic decays of the kaons. These decay amplitudes are

$$\langle\pi^-\mu^+\nu_\mu|T|K^0\rangle = A, \quad (41)$$

$$\langle\pi^+\mu^-\bar{\nu}_\mu|T|\bar{K}^0\rangle = A^*. \quad (42)$$

The kaons propagating to the right from the decay point have to traverse a regenerator made of two layers of different materials  $a$  and  $b$ . On the other hand, the kaons that propagate to the left must traverse a similar regenerator with two layers of the same width but in reverse order,  $b$  followed by  $a$ . With this setting one can show that the amplitude to detect in coincidence two  $\mu^+$  on both detectors is

$$\begin{aligned} A(\pi^-, \pi^-) = & \frac{A^2}{\sqrt{2}}(\langle K^0|U_{ab}|K^0\rangle\langle K^0|U_{ba}-U_{ab}|\bar{K}^0\rangle \\ & + x[\langle K^0|U_{ba}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}|K^0\rangle \\ & - \langle\bar{K}^0|U_{ba}|K^0\rangle\langle K^0|U_{ab}|\bar{K}^0\rangle]). \end{aligned} \quad (43)$$

We have introduced the ratio

$$x = \frac{\langle\pi^-\mu^+\nu_\mu|T|\bar{K}^0\rangle}{\langle\pi^-\mu^+\nu_\mu|T|K^0\rangle} \quad (44)$$

between the  $\Delta S = \Delta Q$  violating amplitude and the dominant one. Experimentally,  $x = [-2 \pm 6 + i(1.2 \pm 1.9)] \times 10^{-3}$  [5], which is consistent with zero; theoretically, within the standard model one expects  $x \sim 10^{-7}$ . Therefore, in Eq. (43) we have neglected the term of order  $x^2$ . Finally, let us point out that Eq. (43) goes trivially to zero when  $a = b$ . This is a simple consequence of the antisymmetry of the initial state. With a similar notation one can obtain the

amplitude for two  $\mu^-$  in coincidence. The result is

$$\begin{aligned} A(\pi^+, \pi^+) = & \frac{(A^*)^2}{\sqrt{2}}(\langle\bar{K}^0|U_{ab}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}-U_{ba}|K^0\rangle \\ & + x^*[\langle K^0|U_{ba}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}|K^0\rangle \\ & - \langle\bar{K}^0|U_{ba}|K^0\rangle\langle K^0|U_{ab}|\bar{K}^0\rangle]). \end{aligned} \quad (45)$$

The amplitude for a  $\mu^+$  in the left detector and a  $\mu^-$  in the right detector is

$$\begin{aligned} A(\pi^-, \pi^+) = & \frac{AA^*}{\sqrt{2}}(\langle K^0|U_{ba}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}|K^0\rangle \\ & - \langle K^0|U_{ba}|K^0\rangle\langle\bar{K}^0|U_{ab}|\bar{K}^0\rangle \\ & + x^*\langle K^0|U_{ab}|K^0\rangle\langle K^0|U_{ba}-U_{ab}|\bar{K}^0\rangle \\ & + x\langle\bar{K}^0|U_{ab}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}-U_{ba}|K^0\rangle). \end{aligned} \quad (46)$$

The other asymmetric amplitude  $A(\pi^+, \pi^-)$  is

$$\begin{aligned} A(\pi^+, \pi^-) = & \frac{AA^*}{\sqrt{2}}(\langle\bar{K}^0|U_{ba}|\bar{K}^0\rangle\langle K^0|U_{ab}|K^0\rangle \\ & - \langle\bar{K}^0|U_{ba}|K^0\rangle\langle K^0|U_{ab}|\bar{K}^0\rangle \\ & + x^*\langle K^0|U_{ab}|K^0\rangle\langle K^0|U_{ba}-U_{ab}|\bar{K}^0\rangle \\ & + x\langle\bar{K}^0|U_{ab}|\bar{K}^0\rangle\langle\bar{K}^0|U_{ab}-U_{ba}|K^0\rangle). \end{aligned} \quad (47)$$

We shall now consider two asymmetries which can be measured in the two-arm experiments,

$$R_1 = \frac{|A(\pi^-, \pi^-)|^2 - |A(\pi^+, \pi^+)|^2}{|A(\pi^-, \pi^+)|^2}, \quad (48)$$

$$R_2 = \frac{|A(\pi^-, \pi^+)|^2 - |A(\pi^+, \pi^-)|^2}{|A(\pi^-, \pi^+)|^2 + |A(\pi^+, \pi^-)|^2}, \quad (49)$$

and also an asymmetry which can be measured in the single arm experiments of the CPLEAR type (see, e.g., Ref. [9]),

$$R_3 = \frac{|A_{ab}(\pi^-)|^2 - |A_{ba}(\pi^+)|^2}{|A_{ab}(\pi^-)|^2 + |A_{ba}(\pi^+)|^2}. \quad (50)$$

Here

$$A_{ab}(\pi^-) = A[\langle K^0|U_{ab}|\bar{K}^0\rangle + x\langle\bar{K}^0|U_{ab}|\bar{K}^0\rangle], \quad (51)$$

$$A_{ba}(\pi^+) = A^*[\langle\bar{K}^0|U_{ba}|K^0\rangle + x^*\langle K^0|U_{ba}|K^0\rangle]. \quad (52)$$

The ratios  $R_1$  and  $R_3$  are CP-asymmetric observables. They depend on the intrinsic (i.e., fundamental) CP violation parameter  $\delta$ . Furthermore, since the regenerators are made out of matter and not of equal amounts of matter and antimatter, they are themselves CP-asymmetric and so induce a macroscopic, extrinsic CP violation which in general contributes to both CP-violating observables,  $R_1$  and  $R_3$ . However, interchanging the order of the layers leads to a partial cancellation of the extrinsic CP violating effects. For this reason the ratio  $R_3$  is primarily sensitive to the fundamental CP violation. In the limit  $x = 0$  the cancellation of the extrinsic CP violation in  $R_3$  is exact and this leads to the result  $R_3 \simeq 2\delta$ . This is not so for  $R_1$  which does not vanish when  $\delta = 0$ . The ratio  $R_1$ , for example, is normally of the order of unity as the extrinsic CP violation is of this order. For an aluminum–tungsten regenerator, using  $x = 0$ ,  $t_a = 24 \times 10^{-11}$  s and  $t_b = 12 \times 10^{-11}$  s, we find  $R_1 = 1.349$  for  $\delta$  given in Ref. [5], whereas for  $\delta = 0$  the corresponding value is  $R_1 = 1.334$ . One should notice that  $R_1$  is very sensitive to the imaginary parts of the effective Hamiltonian. Switching off the imaginary parts of the matter-induced potentials  $V_i$  and  $\bar{V}_i$  reduces  $R_1$  by about a factor of 200, while switching off the decay rates  $\Gamma_S$  and  $\Gamma_L$   $R_1$  would reduce it by about a factor of 8. If all the imaginary parts are set equal to zero,  $R_1$  is suppressed by a factor  $2 \times 10^{-4}$ .

The parameter  $R_2$  may appear as a CP-violating observable too, but in fact it is not. To see that one has to notice that CP transformation not only interchanges particles with their antiparticles but also flips the sign of all the momenta; for the two-arm setup under discussion this implies an additional interchange of the arguments of  $A(\pi_i, \pi_j)$  so that  $R_2$  is unchanged under the CP transformation  $\delta \rightarrow -\delta$ . It has a moderate sensitivity to the imaginary parts of the effective Hamiltonian. For the same regenerator and  $x = 0$ , we find  $R_2 = -0.69$  with normal values of all the imaginary parts. Switching off  $V_i$  and  $\bar{V}_i$  reduces  $|R_2|$  by about a factor of 2, while switching off  $\Gamma_S$  and  $\Gamma_L$  would reduce it by about a factor of 1.4. If all the imaginary parts are set equal to zero,  $R_2$  goes to zero.

Thus, by measuring  $R_1$  and  $R_2$  one can obtain an information on the imaginary parts of the effective Hamiltonian of the  $K^0\bar{K}^0$  system in matter, and in particular on the imaginary parts of the  $K^0(\bar{K}^0)$ –nucleon scattering amplitudes.

In conclusion, we have studied the effects of periodic matter in kaon regeneration. Motivated by the possibility of the parametric resonance in neutrino oscillations in matter we considered similar effects in  $K_L \rightarrow K_S$  transitions. Unfortunately, the large  $\Delta\Gamma$  and imaginary parts of the forward kaon–nucleon scattering amplitudes prevent a sizable enhancement of the  $K_L \rightarrow K_S$  transition probability (cf. Fig. 2). However, some interesting effects can be produced using regenerators made of alternating layers of two different materials. Despite the fact that the regenerator has a fixed length one can obtain different values for the probability distribution of the  $K_L$  decay into a final state (cf. Fig. 3 and Fig. 4). Finally, we have pointed out that using a two-arm regenerator set up it is possible to measure the imaginary parts of the  $K^0(\bar{K}^0)$ –nucleon scattering amplitudes in the correlated decays of the  $\phi$ -resonance. Combining the data of the single-arm regenerator experiments with direct and reverse orders of the matter layers in the regenerator one can independently measure the CP violating parameter  $\delta$ .

## Acknowledgements

We would like to thank J.P. Silva for useful discussions and for a careful reading of the manuscript. The work of E.A. was supported by the “Sonderforschungsbereich 375 für Astro-Teilchenphysik der Deutschen Forschungsgemeinschaft”. P.K. wishes to thank the Jenny and Antti Wihuri foundation and the Finnish Academy of Science and Letters for financial support. This work was supported by Fundação para a Ciência e a Tecnologia through the grants PRAXIS XXI/BPD/20182/99 and CERN/P/FIS/15183/99.

## References

- [1] Q.Y. Liu, A.Yu. Smirnov, Nucl. Phys. B 524 (1998) 505; Q.Y. Liu, S.P. Mikheyev, A.Yu. Smirnov, Phys. Lett. B 440 (1998) 319; S.T. Petcov, Phys. Lett. B 434 (1998) 321; E.Kh. Akhmedov, Nucl. Phys. B 538 (1999) 25; E.Kh. Akhmedov, A. Dighe, P. Lipari, A.Yu. Smirnov, Nucl. Phys. B 542 (1999) 3.
- [2] V.K. Ermilova, V.A. Tsarev, V.A. Chechin, Kr. Soob. Fiz. 5 (1986) 26 (Short Notices of the Lebedev Institute).
- [3] E.Kh. Akhmedov, Yad. Fiz. 47 (1988) 475, Sov. J. Nucl. Phys. 47 (1988) 301.

- [4] G. Castelo Branco, L. Lavoura, J.P. Silva, CP Violation, Oxford Univ. Press, Oxford, 1999.
- [5] D.E. Groom et al., Eur. Phys. J. C 15 (2000) 1.
- [6] J.P. Silva, Phys. Rev. D 62 (2000) 11600P.
- [7] E.Kh. Akhmedov, Nucl. Phys. B 538 (1999) 25.
- [8] R. Baldini, A. Nichetti,  $K_L$  Interactions and  $K_S$  Regeneration in KLOE, LNF-96-008-IR, 1996.
- [9] A. Benelli, Nucl. Phys. Proc. Suppl. B 75 (1999) 267.